Derivation of the Muon g-2 Anomaly from Non-Equilibrium Dynamics

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Abstract

As it is known, the anomalous magnetic moment of muons is a longstanding puzzle of the Standard Model (SM). Commonly referred to as the muon "g-2 problem", the precise testing of this anomaly provides a sensitive probe for physics beyond SM. Here we show that the leading order contribution to the muon anomaly can be estimated from the onset of non-equilibrium dynamics near the Fermi scale. The derivation is straightforward and evades the postulated existence of new phenomena in the low to mid TeV range of high-energy physics.

Key words: g-2 anomaly, Standard Model, minimal fractal manifold, Fractional Field Theory, non-equilibrium dynamics.

1. Introduction

In Quantum Mechanics, a charged particle of mass m carries a magnetic dipole $\boldsymbol{\mu}$ linked with its spin vector $\boldsymbol{s}$ through the following relationship

$$\boldsymbol{\mu} = g\,(\frac{\pm e}{2m})\boldsymbol{s} \tag{1}$$

in which g is the gyromagnetic ratio and e the electric charge. Whereas in Quantum Mechanics g holds a well-defined value ($g = 2$), the cumulative contribution of quantum fluctuations in the SM yields a minute but non-vanishing correction to the gyromagnetic ratio. This correction is parameterized by the so-called *anomalous magnetic moment,* which is defined as

$$a = \frac{g-2}{2} \tag{2}$$

Another useful parameterization of (2) is the deviation between its experimental and theoretical values, i.e.,

$$\Delta a = \left| a^{\exp} - a^{th} \right| \tag{3}$$

When applied to the family of SM charged leptons (the electron e , muon μ and the tau-lepton τ), (3) can be measured with various degrees of accuracy and evaluated against all perturbative corrections demanded by the theory. The overall contribution of the SM may be split into three components, namely,

$$a_l^{SM} = a_l^{QED} + a_l^{EW} + a_l^{Had} \quad , \quad l = \{e, \mu, \tau\} \tag{4}$$

corresponding to the electrodynamics (QED), electroweak (EW) and the hadronic sectors, respectively. The leptonic deviation (3) assumes the form

$$\Delta a_l = \left| a_l^{\exp} - a_l^{SM} \right| \tag{5}$$

Conventional wisdom holds that (5) can be tracked down to either an experimental error, an incorrect computation of hadronic loop-diagrams or a hint for "*new physics*" phenomena (NP), alleged to surface beyond the energy range of the SM. While the electron anomalous moment Δa_e is relatively insensitive to the EW and strong sectors of the SM, the muon anomalous moment Δa_μ contains contributions from all SM sectors. As a result, precise tests of Δa_μ are believed to provide an excellent opportunity to reveal or constrain the signature of NP, including, for example, supersymmetry, leptoquarks, extended gauge groups, seesaw models, extra dimensions, extended Higgs sectors and other field theories beyond SM (referred below to BSM scenarios) [1-3].

The most recent experimental values on the magnitude of Δa_e and Δa_μ are [1, 7]

$$\Delta a_e^{\exp} = (-0.88) \times 10^{-12} \tag{6}$$

$$\Delta a_\mu^{\exp} = 2.87(\pm 0.80) \times 10^{-9} \tag{7}$$

The study of lepton magnetic moments represents an active research subject in particle physics. For additional information and in-depth details, the reader is directed to the large database of articles and books devoted to this topic.

2. Non-equilibrium dynamics as source of the g-2 anomaly

The concept of *helicity* in Quantum Field Theory (QFT) refers to the projection of spin onto the direction of the momentum vector. A distinctive feature of the anomalous magnetic moment of leptons is that it can flip helicity in interactions mediated by gauge bosons [2]. These flip transitions are only allowed for massive particles with a probability amplitude proportional to the mass of the particle. Since the probability of such helicity flips scales with the square of the probability amplitude, the lepton anomalous magnetic moment Δa_l must scale with the square of the lepton mass. In fact, it can be shown that, in the presence of BSM physics, the leading one-loop contribution to the muon anomalous magnetic Δa_{μ} is on the order of [1]

$$\Delta a_{\mu} \sim \frac{g_{NP}^{2}}{16\pi^{2}} \frac{m_{\mu}^{2}}{M_{NP}^{2}} \tag{8}$$

where g_{NP} and M_{NP} denote the coupling and mass associated with the onset of the NP sector, in particular*, heavy BSM particles* alleged to show up beyond the energy range of the Large Hadron Collider. Many avenues centered around (8) have been explored from various vantage points. For instance, [1] examines the possibility of one or two new fields with different spins and gauge group representations, under the hypothesis of weakly coupling $|g_{NP}| \leq \sqrt{4\pi}$ and weak-scale masses $M_{NP} \geq 100\,\text{GeV}$.

In what follows we continue to use (8) as a baseline but interpret it in a new light. Specifically, our analysis proceeds with the following assumptions:

1) Rather than postulating that (8) follows from the NP sector of particle physics, we conjecture that (8) is rooted in the onset of *non-equilibrium dynamics near* the Fermi scale given by $M_{NP} \rightarrow M_{EW} \approx O(246\,\text{GeV})$ [4-6].

2) we consider the dominant muon decay channel known as the *Michel decay* [2],

$$\mu^{-} \rightarrow \nu_{\mu} e^{-} \overline{\nu_{e}} \tag{9}$$

which is a pure leptonic process mediated by the weak boson W^{-}. As such, (9) is a reliable process since it has high-statistics, it enables precise measurements of the Fermi constant (G_F) and it accounts for nearly 100% of the branching ratio.

Because we are limiting the discussion to the Michel decay, it makes sense to approximate the scale where non-equilibrium sets in as $O(M_{EW}) \approx M_W$. This is to say that the mass of the mediating W^- boson marks the representative non-equilibrium scale for (9).

The muon coupling in (9) can be extracted from the SM relationship

$$\frac{G_F}{\sqrt{2}} = \frac{g_\mu^2}{8M_W^2} \tag{10}$$

where M_W is the mass of the W boson. From (8) and (10) we derive

$$\Delta a_\mu \sim \frac{G_F m_\mu^2}{2\sqrt{2}\pi^2} \tag{11}$$

which yields

$$\boxed{\Delta a_\mu \sim 4.667 \times 10^{-9}} \tag{12}$$

in promising agreement with the experimental data (7).

A natural follow-up question is the following: Would the same analysis apply to the anomalous magnetic moments of electrons and tau-leptons? Assuming that this line of thinking is correct and that the muon coupling g_μ stays the same for electrons and tau-leptons, we use (8) to estimate the non-equilibrium mass scales that match existing data on Δa_e and Δa_μ [1-3, 7-8]. Numerical evaluation leads to

$$\Delta a_e \to \lambda M_W, \lambda = 1/3 \tag{13}$$

$$\Delta a_{\tau} \to \lambda M_{W} \approx M_{EW}, \lambda = 3 \tag{14}$$

It is also instructive to ponder on the possible corrections to the muon coupling due to the "would-be" contribution of lepton flavor violation (LFV) [2]. This analysis goes beyond the scope of the paper and may be considered elsewhere.

Our findings are summarized in the table below.

	Experiment	**Prediction**
Δa_{μ}	$2.87(\pm 0.80) \times 10^{-9}$	4.667×10^{-9}
Δa_{e}	$(-0.88) \times 10^{-12}$	$(-0.605) \times 10^{-12}$ for $O(M_{EW}) = (1/3) M_{W}$
Δa_{τ}	$-0.0508 < \Delta a_{\tau}^{\exp} < 0.011823$	$\sim 10^{-7}$ for $O(M_{EW}) = 3M_{W}$

Table 1: Comparison between experimental and predicted values of Δa_{l}

References

[1] https://arxiv.org/pdf/1402.7065.pdf

[2] https://arxiv.org/pdf/1610.06587.pdf

[3] https://arxiv.org/pdf/2002.04822.pdf

[4] Available at the following sites:

http://www.aracneeditrice.it/aracneweb/index.php/pubblicazione.html?item=9788854889972

https://www.researchgate.net/publication/278849474_Introduction_to_Fractional_Field_Theory_consolidated_version

[5] http://www.ejtp.info/articles/ejtpv7i24p219.pdf

[6] Available at the following site:

https://www.researchgate.net/publication/266578650_REFLECTIONS_ON_THE_FUTURE_OF_PARTICLE_THEORY

[7] https://science.sciencemag.org/content/360/6385/191

[8] https://iopscience.iop.org/article/10.1088/1742-6596/912/1/012001/pdf